Fermionic Zero Modes in Self-dual Vortex Background

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Abstract

We study fermionic zero modes in the background of self-dual vortex on a two-dimensional non-compact extra space in 5+1 dimensions. In the Abelian Higgs model, we present an unified description of the topological and non-topological self-dual vortex on the extra two dimensions. Based on it, we study localization of bulk fermions on a brane with inclusion of Yang-Mills and gravity backgrounds in six dimensions. Through two simple cases, it is shown that the vortex background contributes a phase shift to the fermionic zero mode, this phase is actually origin from the Aharonov-Bohm effect.

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I. INTRODUCTION

In four dimensional space time, the interactions of fermions in a Nielsen-Olesen vortex background have been widely analyzed in the literature, mainly in connection with bound states at threshold[1], zero modes[2] and scattering solutions[3]. Recently, Frere, Libanov and Troitsky have shown that a single family of fermions in six dimensions with vector-like couplings to the Standard Model (SM) bosons gives rise to three generations of chiral Standard Model fermions in four dimensions[4, 5]. In 5+1 dimensions, Frere *et al* also studied the fermionic zero modes in the background of a vortex-like solution on an extra two-dimensional sphere and relate them to the replication of fermion families in the Standard Model[6].

The topological vortex (especially Abrikosov-Nielsen-Olesen vortex) coupled to fermions may lead to chiral fermionic zero modes[7]. Usually the number of the zero modes coincides with the topological number, that is, with the magnetic flux of the vortex. In Large Extra Dimensions (LED) models, the chiral fermions of the Standard Model are described by the zero modes of multi-dimensional fermions localized in the (four-dimensional) core of a topological defect. Unlike the classical Kaluza Klein theory where one assumes that the extra dimensions should be small and cover a compact manifold, the extra dimensions can be large and non compact[8, 9]. This freedom can provide new insights for a solution of gauge hierarchy problem[10] and cosmological constant problem. While we shall study fermionic zero modes coupled with a self-dual vortex background[11] on a two dimensional non-compact extra space in 5+1 dimensions.

The paper is organized as follows: In section II, we first present the unified description of the topological and non-topological self-dual vortex in Abelian Higgs model on the two dimensional non-compact extra space. In section III, In 5+1 dimensions, we analyzed the effective lagrangian of the fermions localized on a brane in the background of the coupling between Higgs field and fermion spinor field. In section IV, two simple cases are discussed to show the role of vortex background in the fermionic zero modes. In the last section, a brief conclusion is presented.

II. SELF-DUAL VORTEX ON A TWO-DIMENSIONAL NON-COMPACT EXTRA SPACE

We consider a 5+1 dimensional space-time $M^4 \times R^2$ with M^4 represents our four-dimensional space-time and R^2 represents the two-dimensional extra Euclidean space. The metric G_{MN} of the manifold $M^4 \times R^2$ is

$$ds^2 = G_{MN}dx^M dx^N = g_{\mu\nu}dx^\mu dx^\nu - \delta_{ij}dx^i dx^j, \tag{1}$$

where $g_{\mu\nu}$ is the four dimensional metric of the manifold M^4, capital Latin indices $M, N = 0, \cdots, 5$, Greek indices $\mu, \nu = 0, \cdots, 3$, lower Latin indices $i, j = 4, 5$ and x^4, x^5 are the coordinates on R^2. To generate the vortex solution, we introduce the Abelian Higgs Lagrangian

$$\mathcal{L}_V = \sqrt{-G}\left(-\frac{1}{4}F_{MN}F^{MN} + (D^M\phi)^\dagger(D_M\phi) - \frac{\lambda}{2}(\|\phi\|^2 - v^2)^2\right), \tag{2}$$

where $G = \det(G_{MN})$, $F_{MN} = \partial_M A_N - \partial_N A_M$, $D_M\phi = (\partial_M - ieA_M)\phi$, $\phi = \phi(x^4, x^5)$ and A_M are a complex scalar field on R^2 and a U(1) gauge field, respectively, $\|\phi\| = (\phi\phi^*)^{\frac{1}{2}}$.

The Abrikosov-Neilsen-Olsen vortex solution on the $M^4 \times R^2$ could be generated from the Higgs field. We first introduce the first-order Bogomol'nyi self-dual equations [12] on the two dimensional extra Euclidean space

$$\begin{aligned} D_\pm\phi = 0, \quad B &= \partial_i\partial_i \ln(\|\phi\|^2) \\ &= \pm e(\|\phi\|^2 - v^2), (i = 4, 5) \end{aligned} \tag{3}$$

here the operator $D_\pm$ is defined as $D_\pm \equiv (D_4 \pm iD_5)$. We know that complex Higgs field ϕ can be regarded as the complex representation of a two-dimensional vector field $\vec{\phi} = (\phi^1, \phi^2)$ over the base space time, it is actually a section of a complex line bundle on the base manifold. Considering the self-dual equation $D_+\phi = 0$ and $\phi = \phi^1 + i\phi^2$, we split the real part form the imaginary part, and obtain two equations

$$\begin{aligned} \partial_4\phi^1 - \partial_5\phi^2 &= eA_4\phi^2 + eA_5\phi^1, \\ \partial_4\phi^2 + \partial_5\phi^1 &= eA_5\phi^2 - eA_4\phi^1. \end{aligned} \tag{4}$$

Substituting Eqs. (4) and $\phi = \phi^1 + i\phi^2$ into $\partial_4\phi^*\phi - \partial_4\phi\phi^*$, it is easy to verify

$$\partial_4\phi^*\phi - \partial_4\phi\phi^* = 2ieA_4\|\phi\|^2 + i(\partial_5\phi^*\phi + \partial_5\phi\phi^*). \tag{5}$$

Considering the fundamental identity

$$\epsilon_{ab}n^a\partial_i n^b = \frac{1}{2i}\frac{1}{\phi\phi^*}(\partial_i\phi^*\phi - \partial_i\phi\phi^*) \tag{6}$$

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with the unit vector defined as $n^a = \phi^a / \|\phi\|, (a, b = 1, 2)$, we immediately have

$$eA_4 = \epsilon_{ab} n^a \partial_4 n^b - \frac{1}{2} \partial_5 \ln(\phi\phi^*). \tag{7}$$

When considering $\partial_5 \phi^* \phi - \partial_5 \phi \phi^*$, following the same process above, it yields

$$eA_5 = \epsilon_{ab} n^a \partial_5 n^b + \frac{1}{2} \partial_4 \ln(\phi\phi^*). \tag{8}$$

Eq. (7) and Eq. (8) can be unified into one equation

$$eA_i = \epsilon_{ab} n^a \partial_i n^b - \frac{1}{2} \epsilon_{ij} \partial_j \ln(\phi\phi^*). \tag{9}$$

Repeating the same discussion above to $D_- \phi = 0$, we arrive

$$eA_i = \epsilon_{ab} n^a \partial_i n^b + \frac{1}{2} \epsilon_{ij} \partial_j \ln(\phi\phi^*). \tag{10}$$

In fact, since the magnetic field is $B = \epsilon^{ij} \partial_i (eA_j)$, according to Eq. (10), we have

$$B = \epsilon^{ij} \epsilon_{ab} \partial_i n^a \partial_j n^b + \partial_i \partial_i \ln(\|\phi\|^2).$$

So the second one in the Bogomol'nyi self-dual equation (3) can be generalized to

$$B = \epsilon^{ij} \epsilon_{ab} \partial_i n^a \partial_j n^b + \partial_i \partial_i \ln(\|\phi\|^2) = \pm e(\|\phi\|^2 - v^2). \tag{11}$$

For clarity we denote

$$\begin{aligned} B &= B_T + B_{NT}, \\ B_T &= \epsilon^{ij} \epsilon_{ab} \partial_i n^a \partial_j n^b, \\ B_{NT} &= \partial_i \partial_i \ln(\|\phi\|^2). \end{aligned} \tag{12}$$

According to Duan's topological current theory, it is easy to see that the first term of Eq. (11) bear a topological origin. From Duan's ϕ-mapping topological current theory [13], one can see that the topological term of the magnetic field B_T

$$B_T = \epsilon^{ij} \epsilon_{ab} \partial_i n^a \partial_j n^b, \tag{13}$$

just describes the non-trivial distribution of $\vec{n}$ at large distances in space [14]. Noticing $\partial_i n^a = \frac{\partial_i \phi^a}{\|\phi\|} + \phi^a \partial_i \frac{1}{\|\phi\|}$ and the Green function relation in ϕ−space : $\partial_a \partial_a ln(\|\phi\|) = 2\pi\delta^2(\vec{\phi}), (\partial_a = \frac{\partial}{\partial \phi^a})$, it can be proved that [15]

$$B_T = \delta^2(\phi) J(\frac{\phi}{x}). \tag{14}$$

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So the second one in the Bogomol'nyi self-dual equations (3) should be

$$B = \delta^2(\phi) J(\frac{\phi}{x}) + \partial_i \partial_i \ln \|\phi\|^2 = \pm e(\|\phi\|^2 - v^2). \tag{15}$$

This equation is more exact than the conventional self-dual equation, in which the topological term has been ignored all the time. Obviously when the field $\phi \neq 0$, the topological term vanishes and we have

$$B = B_{NT} = \partial_i \partial_i \ln(\|\phi\|^2). \tag{16}$$

So the self-dual equation (15) reduces to a nonlinear elliptic equation for the scalar field density $\|\phi\|^2$

$$\partial_i \partial_i \ln(\|\phi\|^2) = \pm e(\|\phi\|^2 - v^2). \tag{17}$$

This is just the conventional self-dual equation. Comparing this equation with Eq. (15), one see that the topological term $\delta^2(\vec{\phi}) J(\frac{\phi}{x})$ is missed. The exact self-dual equation should be Eq. (15). From our previous work, obviously the first term of Eq. (15) describes the topological self-dual vortices. As for conventional self-dual nonlinear equation (17), a great deal of work has been done by many physicists on it, and a vortex-like solutions was given by A. Jaffe [16]. But no exact solutions are known.

Now we see that there are two classes of vortex which arise correspondingly from the symmetric phase and asymmetric phase of the Higgs field. These two classes of vortex provide different vortex background. And we shall study fermionic zero modes coupled with the vortex background in the following two sections.

III. FERMIONIC ZERO MODES IN THE VORTEX BACKGROUND

The lagrangian of the fermions in the vortex background on R^2 is

$$\mathcal{L} = \sqrt{-G} \left\{ \bar{\Psi} \Gamma^A E_A^M (\partial_M - \Omega_M + A_M) \Psi - g\phi \Psi^\dagger \Psi \right\}. \tag{18}$$

where E_A^M is the *sechsbein* with

$$E_A^M = (e_A^\mu, \delta_A^4, \delta_A^5) \tag{19}$$

and capital Latin indices $A, B = 0, \cdots, 5$ correspond to the flat tangent six-dimensional Minkowski space, $\Omega_M = \frac{1}{2} \Omega_M^{AB} I_{AB}$ is the spin connection with the following representation of six-dimensional 8×8 Dirac matrices Γ^A:

$$\Gamma^A = \begin{pmatrix} 0 & \Sigma^A \\ \bar{\Sigma}^A & 0 \end{pmatrix}, \tag{20}$$

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where $\Sigma^0 = \bar{\Sigma}^0 = \gamma^0\gamma^0$; $\Sigma^k = -\bar{\Sigma}^k = \gamma^0\gamma^k(k = 1, 2, 3)$; $\Sigma^4 = -\bar{\Sigma}^4 = i\gamma^0\gamma^5$; $\Sigma^5 = -\bar{\Sigma}^5 = \gamma^0$, γ^μ and γ^5 are usual four-dimensional Dirac matrices in the chiral representation:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix}, \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{21}$$

here σ^k are the Pauli matrices. Γ^A follows relation $\Gamma^A\Gamma^B + \Gamma^B\Gamma^A = 2\eta^{AB}I$, in which $\eta^{AB} = \mathrm{diag}(+, -, \cdots, -)$ is the six-dimensional Minkowski metric.

The components of Ω_M are

$$\Omega_\mu = \omega_\mu, \quad \Omega_4 = 0, \quad \Omega_5 = 0, \tag{22}$$

where $\omega_\mu = \frac{1}{2}\omega_\mu^{ab}I_{ab}$ is the spin connection derived from the metric $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$, lower case Latin indices $a, b = 0, \cdots, 3$ correspond to the flat tangent four-dimensional Minkowski space.

Using Eqs. (22), the lagrangian (18) of the fermions then becomes

$$\mathcal{L} = \sqrt{-G}\{\bar{\Psi}\Gamma^a e_a^\mu(\partial_\mu - \omega_\mu + A_\mu)\Psi + \bar{\Psi}\Gamma^4(\partial_4 + A_4)\Psi + \bar{\Psi}\Gamma^5(\partial_5 + A_5)\Psi - g\phi\Psi^\dagger\Psi\}. \tag{23}$$

We denote the Dirac operator on R^2 with D_R:

$$D_R = \bar{\Gamma}\left\{\Gamma^4(\partial_4 + A_4) + \Gamma^5(\partial_5 + A_5) - g\phi\right\}, \tag{24}$$

where $\bar{\Gamma} = \Gamma^0\Gamma^1\Gamma^2\Gamma^3$, and expand any spinor $\Psi(x^\mu, x^i)$ in a set of eigenvectors $\Theta_m(x^i)$ of this operator D_R

$$D_R\Theta_m(x^i) = \lambda_m\Theta_m(x^i) \quad (i = 4, 5). \tag{25}$$

There may exist a set of discrete eigenvalues λ_m with some separation. All these eigenvalues play a role of the mass of the corresponding four-dimensional excitations [4]. We assume that the energy scales probed by a four-dimensional observer are smaller than the separation, and thus even the first non-zero level is not excited. So, we are interested only in the zero modes of D_R:

$$D_R\Theta(x^i) = 0. \tag{26}$$

This is just the Dirac equation on R^2 with gauge and vortex backgrounds. For fermionic zero modes, we can write

$$\Psi(x^\mu, x^i) = \psi(x)\Theta(x^i), \tag{27}$$

where ψ and Θ satisfy

$$\begin{aligned} \bar{\Gamma}\Gamma^a e_a^\mu(\partial_\mu - \omega_\mu + A_\mu)\psi(x) &= 0, \\ D_R\Theta(x^i) &= 0. \end{aligned} \tag{28}$$

The effective Lagrangian for ψ then becomes

$$\int dx^4 dx^5 \sqrt{-G} \left\{ \bar{\Psi} \Gamma^A E_A^M (\partial_M - \Omega_M + A_M)\Psi - g\phi\Psi^\dagger\Psi \right\}$$
$$= \sqrt{-\det(g_{\mu\nu})} \bar{\psi} \Gamma^a e_a^\mu (\partial_\mu - \omega_\mu + A_\mu)\psi \int dx^4 dx^5 \Theta^\dagger\Theta. \tag{29}$$

Thus, to have the localization of gravity and finite kinetic energy for ψ, the above integral must be finite. This may be achieved for some $\Theta(x^i)$ which does not diverge on the whole R^2 and converge to zero as r tends to infinity.

IV. SIMPLE SITUATIONS

In this section, to illustrate how the vortex background affects the fermionic zero mode, we first discuss the simple case that the Higgs field ϕ is relative only to x^4, and then solve the general Dirac equation for the vacuum Higgs field solution $\|\phi\|^2 = v^2$.

A. Case I: ϕ is relative only to x^4

Now we discuss a simple situation for ϕ, i.e. $\phi = \phi(x^4) = \phi^1(x^4) + i\phi^2(x^4)$. In this case, Eq. (7) and Eq. (8) can be written as

$$A_4 = \frac{1}{e\|\phi\|^2} \left(\epsilon_{ab}\phi^a \partial_4 \phi^b - \frac{1}{\|\phi\|^2}\epsilon_{ab}\phi^a \phi^b \phi^c \partial_4 \phi^c \right), \tag{30}$$

$$A_5 = -\frac{1}{2e}\partial_4 \ln\|\phi\|^2. \tag{31}$$

Then the Dirac operator D_R becomes:

$$D_R = \bar{\Gamma} \left\{ \Gamma^4 \left[\partial_4 + A_4(x^4) - \Gamma^4\Gamma^5 A_5(x^4) + \Gamma^4 g\phi(x^4) \right] + \Gamma^5 \partial_5 \right\}, \tag{32}$$

and Dirac equation $D_R\Theta(x^4, x^5) = 0$ is

$$\left\{ \bar{\Gamma}\Gamma^4 \left[\partial_4 + A_4(x^4) - \Gamma^4\Gamma^5 A_5(x^4) + \Gamma^4 g\phi(x^4) \right] + \bar{\Gamma}\Gamma^5 \partial_5 \right\} \Theta(x^4, x^5) = 0. \tag{33}$$

Here $\Theta(x^4, x^5)$ can be written as the following form:

$$\Theta(x^4, x^5) = f(x^4)h(x^5), \tag{34}$$

where $h(x^5) = Const$ and $f(x^4)$ satisfies

$$\left\{ \partial_4 + A_4(x^4) - \Gamma^4\Gamma^5 A_5(x^4) + \Gamma^4 g\phi(x^4) \right\} f(x^4) = 0. \tag{35}$$

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Solving this equation, one can easily obtain the formalized solution:

$$f(x^4) = e^{-\int dx^4 \left\{ A_4(x^4) - i\gamma^5 A_5(x^4) + g\Gamma^4 \phi(x^4) \right\}}. \tag{36}$$

This equation spontaneously leads to the Aharonov-Bohm phase. Considering $\phi = \phi(x^4)$ and integrating over the extra dimensions for the Eq. (15), one can get

$$\partial_4 \ln \|\phi\|^2 = -\sum_l W_l \pm \int dx^4 (\|\phi\|^2 - v^2), \tag{37}$$

where W_l is winding number. Making use of Eq. (31) and substituting Eq. (37) into Eq. (36), we get the following form

$$f(x^4) = e^{\frac{i\gamma^5}{2e}\sum_l W_l - \int dx^4 \left\{ A_4(x^4) \pm \frac{i\gamma^5}{2e} \int dx^4 (\|\phi\|^2 - v^2) + g\Gamma^4 \phi(x^4) \right\}}. \tag{38}$$

From the first term $e^{\frac{i}{2e}\gamma^5 \sum_l W_l}$, we see that the total topological charge $Q = \sum W_l$ contributes a phase factor to the zero mode $\Theta(x^i) = Cf(x^4)$. The topological charge is determined by the topological properties of the extra space manifold. When we add a point in the infinity, the non-compact R^2 can be compactified to a 2-sphere. In this case, the total topological charge is just the Euler characteristic number of 2-shpere, i.e., $Q = 2$. Eq. (15) reveals that this topological phase origin from the symmetric phase of the Higgs field, and the non-topological one arise from asymmetric phase, it is also included in $e^{\pm\frac{i}{2e}\gamma^5 \int dx^4 (\|\phi\|^2 - v^2)}$. So the topological and non-topological self-dual vortex both contribute a phase shift to the fermionic zero mode.

As all known, quantum topological and geometrical phases are ubiquitous in modern physics—in cosmology, particle physics, modern string theory and condensed matter. In fact, according to Eq. (38), we see this phase shift is actually the quantum mechanical Aharonov-Bohm phase. This discission can be generalized to the AB phase of non-abelian gauge theories, such as the Wilson and 't Hooft loops. Since the AB phase is fundamental to theories of anyons and to gauge fields, it is an important tool for studying the issues of confinement and spontaneous symmetry breaking.

B.   Case II: The vacuum solution

For the vacuum solution of Eq. (15) $\|\phi\|^2 = \phi\phi^* = v^2$ which represents a circle S^1 in the extra space, according to Eq. (10), we see the non-topological part $\frac{1}{2}\epsilon_{ij}\partial_j \ln(\phi\phi^*)$ vanishes, there is only topological part left. When the Higgs field is degenerated on the vacuum

manifold, we have $A_4 = A_5 = 0$, then the Dirac equation $D_R\Theta(x^4, x^5) = 0$ is read as

$$\left\{\bar{\Gamma}\Gamma^4\left(\partial_4 + gv\Gamma^4\right) + \bar{\Gamma}\Gamma^5\partial_5\right\}\Theta(x^4, x^5) = 0. \tag{39}$$

In which $\Theta(x^4, x^5) = f(x^4)h(x^5)$, $h(x^5)$ is a constant again and $f(x^4)$ satisfy the following equation

$$\bar{\Gamma}\Gamma^4\left(\partial_4 + gv\Gamma^4\right)f(x^4) = 0. \tag{40}$$

Denoting

$$f(x^4) = \begin{pmatrix} f_1(x^4) \\ f_2(x^4) \\ f_3(x^4) \\ f_4(x^4) \end{pmatrix}, \tag{41}$$

one obtains the following two sets of the differential equations

$$\begin{cases} \partial_4 f_1(x^4) - igv f_4(x^4) = 0, \\ \partial_4 f_4(x^4) - igv f_1(x^4) = 0; \end{cases} \tag{42}$$

$$\begin{cases} \partial_4 f_2(x^4) + igv f_3(x^4) = 0, \\ \partial_4 f_3(x^4) + igv f_2(x^4) = 0. \end{cases} \tag{43}$$

The solutions are

$$\begin{aligned} f_1(x^4) &= C_1 e^{iQx^4} + C_4 e^{-iQx^4}, \\ f_2(x^4) &= C_2 e^{Qx^4} + C_3 e^{-Qx^4}, \\ f_3(x^4) &= i(C_2 e^{Qx^4} - C_3 e^{-Qx^4}), \\ f_4(x^4) &= C_1 e^{iQx^4} - C_4 e^{-iQx^4}, \end{aligned} \tag{44}$$

where $Q = gv$. Now we see that $f_1(x^4)$ and $f_4(x^4)$ are planar wave function. It is easy to see that, if the coupling constant $g = 0$ or the vacuum expectation $v = 0$, the solution $f(x^4)$ is simply a constant spinor. As shown in section II, $\phi = v = 0$ corresponds to the symmetric phase and $\phi = v \neq 0$ corresponds to the asymmetric phase. So different vortex background results in different zero mode.

The discussion above can also be generalized to a more universal case, usually the general Dirac equation is hardly solvable, while the two simple cases above provide us a coarse insight into the fermionic zero modes in the vortex background.

V. CONCLUSION

In 5+1 dimensions, there are two classes of vortex solutions in the Abelian Higgs model: the topological vortex and the non-topological vortex. They can be described by a more accurate Bogomol'nyi self-duality equation $B = \delta^2(\vec{\phi})J(\frac{\phi}{x}) \pm \partial_i\partial_i ln(\|\phi\|^2)$. The topological vortex just arise from the symmetric phase of the Higgs field, while the non-topological vortex origin from the asymmetric phase. Through a simple case, it is shown that the vortex background contribute a phase shift to the fermionic zero mode in the 5+1 dimensional space time. The phase is divided into two parts, one is related with the topological number of the extra space, the other depends on the non-topological vortex solution. Then we solve the general Dirac equation for the vacuum case, the symmetric and asymmetric phases of the Higgs field just correspond to different fermion solutions.

VI. ACKNOWLEDGMENT

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[1] C. R. Nohl, Phys. Rev. D **12**, 1840 (1975).

[2] R. Jackiw, Phys. Rev. D **29**, 2375 (1984).

[3] H. J. de Vega, Phys. Rev. D **18**, 2932 (1978).

[4] M. V. Libanov and S. V. Troitsky, Nucl. Phys. B **599**, 319 (2001).

[5] J. M. Frere, M. V. Libanov and S. V. Troitsky, Phys. Lett. B **512**, 169 (2001); JHEP **0111**, 025 (2001).

[6] J. M. Frere, M. V. Libanov, E. Ya. Nugaev and S. V. Troitsky, JHEP **0306**, 009 (2003).

[7] R. Jackiw and P. Rossi, Nucl. Phys. B **190**, 681 (1981).

[8] V. Rubakov and M. Shaposhnikov, Phys. Lett. B **125**, 139 (1983).

[9] S. Randjbar-Daemi and C. Wetterich, Phys. Lett. B **166**, 65 (1986).

[10] L. Randal and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1997).

[11] Y. Q. Wang, Master thesis, Lanzhou University (2005).

[12] E. Bogomol'nyi, Sov. J. Nucl. Phys. **24**, 449 (1976).

[13] Y. S. Duan, SLAC-PUB-3301 (1984); Y. S. Duan, L. B. Fu and G. Jia, J. Math. Phys. **41**, 4379 (2000).

[14] G. 't Hooft, Nucl. Phys. B **79**, 276 (1974); A. Polyakov, JETP Lett. **20**, 194 (1974).

[15] Y. S. Duan, S. Li and G. H. Yang, Nucl. Phys. B **514**, 705 (1998).

[16] A. Jaffe and C. Taubes, Vortices and Monopoles, (Birkhauser 1980).